Energy gain of injected electrons subjected to an intense laser field and its magnetic field induced in plasma

Guihua Zeng
National Key Laboratory on ISDN, XiDian University, Xi'an 710071, China
(Received 24 July 1998; revised manuscript received 24 May 1999)

Cyclotron-resonance energy gain of injected electrons subjected to an intense circularly polarized laser field and the magnetic field induced in a low-density plasma is investigated theoretically. By considering the inverse Faraday effect (IFE), where a circularly polarized finite area laser beam induces an axial magnetic field in a plasma, it is found that very interesting energy gains can be obtained by Doppler-shifted cyclotron resonance in this field for the appropriate injection velocity. This same IFE field also acts to confine these electrons radially and, on exiting the plasma adiabatically, it is in this way that the transverse electron energy is converted to axial energy. Two limits to the energy gain are discussed: (i) cyclotron radius of the energetic electrons becoming comparable to the beam, and (ii) axial dephasing. [S1063-651X(99)09509-4]

PACS number(s): 52.75.Di, 52.40.Nk, 52.35.Mw

I. INTRODUCTION

The recent advent of extremely high intensity, ultrafast lasers using fiber compression [1], Kerr-lens mode locking [2], and more recently, chirped pulse amplification [3–5], has made it possible to explore a new regime of relativistic laser-plasma interaction. This has stimulated a great interest in the investigation of charge particle acceleration by an intense laser beam [6–18]. Various ideas for electron acceleration may be separated into three kinds of schemes, viz., acceleration (a) by the strong field or the longitudinal ponderomotive force of laser pulse [6–10], (b) by the high-gradient field associated with plasma waves [11–16], and (c) by a strong laser field with weakly external field in vacuum [17,18]. In scheme (a), the particle gains energy and momentum at the expense of the pulse, and the particle is accelerated in half-pulse extent. The most elaborate models of the scheme (b) are the plasma wake-field acceleration, the plasma beat-wave acceleration, and the laser wake-field acceleration, including the self-modulation regime and wake accelerator driven by multiple laser pulse. In these configurations, a high-gradient plasma wave is excited resonantly, and accelerates the particle. In scheme (c), the weakly external field breaks the symmetry of the laser field, so that the electron may be accelerated in the half-wavelength region of the electromagnetic wave. These concepts each have their difficulties and so it is not without interest to consider a new idea. This concept requires the use of circular polarization for the laser pulse since it relies on some features of the well-known inverse Faraday effect that is produced by such a pulse.

The combination of an electromagnetic field of circularly polarized laser light and an IFE magnetic field created by the laser itself can bring about the acceleration of injected electron as in the conventional electron cyclotron-resonance (ECR) scheme. A quasistatic magnetic field is produced by IFE for a circularly polarized laser wave. The effect occurs at the periphery of a wave beam when the latter is not uniform in the transverse direction. The magnetic field is parallel to the direction of wave propagation for the left circular polarization and antiparallel for the right circular one. The injected electron motion will be a combination of circular orbits at the cyclotron frequency and at the Doppler-shifted laser frequency. When these two frequencies are close so the cyclotron-resonance condition is nearly satisfied, the energy of the electrons can be changed dramatically and favorably phased electrons gain considerable energy. Thus the electron is accelerated by the like-ECR mechanism. However, the acceleration will be terminated when the cyclotron radius of the injected electron is larger than the effective spot of laser beam or the injected electron escapes from the laser pulse because of axial dephasing. The trajectory of the injected electron depends on the induced magnetic field. Due to the induced magnetic field dependence on the laser intensities, one can control the electron motion by the laser intensities.

In the new regime of relativistic laser-plasma interaction, the proposed mechanisms for the induced magnetic field include the ponderomotive force mechanism [19–21], the inverse Faraday effect (IFE) [22–25], the ionization front [26], and the vortex dynamics mechanism [27]. Here, we are interested in the IFE mechanism. By taking into account relativistic effects, a relativistic evolution equation for the induced magnetic field is obtained, which in the nonrelativistic limit had a form analogous to that presented in Refs. [20,21]. A numerical solution of the equation obtained gives the variation of the induced magnetic field along and across of the laser beam.

This paper is organized as follows. Due to this model being associated with the induced magnetic field, it is given a detailed discussion in Sec. II. An acceleration scheme is proposed in Sec. III, and an energy evolution equation for the injected electron in the combination field is derived. Results show that the energy of the injected electron increases with time, however the electron spirals outward from the laser beam when the cyclotron radius is sufficiently larger, or the electron escapes from the laser pulse due to the dephasing in the longitudinal direction. In Sec. IV, we analyze the trajectory of the injected electron in the laser beam and the emission angle of the injected electrons. In Sec. V, we remark on this topic and the conclusions are presented.

II. QUASISTATIC MAGNETIC FIELD

Interaction of an ultraintense laser with plasma can be described by Maxwell's equations and relativistic equations that describe the motions of the cold electron fluid,

$$\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=\frac{4\pi}{c}\nabla\times\mathbf{J}_p, \tag{2.1}$$

$$\frac{\partial\mathbf{P}_p}{\partial t}+(\mathbf{V}_p\cdot\nabla)\mathbf{P}_p=-e\left(\mathbf{E}+\frac{\mathbf{V}_p}{c}\times\mathbf{B}\right), \tag{2.2}$$

$$\frac{\partial n}{\partial t}=-\nabla\cdot(n\mathbf{V}_p), \tag{2.3}$$

where $\mathbf{P}_p,\mathbf{V}_p$ are the plasma electrons momentum and velocity, and $\mathbf{J}_p=-en\mathbf{V}_p,\mathbf{J}_p$ is the current density. Letting

$$\mathcal{Q}=\frac{4\pi}{c}\nabla\times\mathbf{J}_p, \tag{2.4}$$

and defining the vector field

$$\mathbf{P}_c=m\gamma\mathbf{V}_p-\frac{e}{c}\mathbf{A}, \tag{2.5}$$

where $\mathbf{A}$ is the vector potential of the laser pulse, $\gamma=\sqrt{1+p_p^2}$ is the relativistic factor. By taking the curl of the plasma electron momentum equation, we obtain

$$\frac{\partial\mathbf{P}_c}{\partial t}=\nabla\times\mathbf{V}_p\times\nabla\times\mathbf{P}_c. \tag{2.6}$$

If $\nabla\times\mathbf{P}_c=\mathbf{0}$ everywhere at $t=0$, $\nabla\times\mathbf{P}_c$ vanishes everywhere for all time [19]. Equation (2.5) gives

$$\frac{e}{c}\mathbf{B}=\nabla\times\mathbf{P}_p. \tag{2.7}$$

In term of Eq. (2.7) and $\mathbf{P}_p=m\gamma\mathbf{V}_p$, we have

$$\nabla\times\mathbf{V}_p=\frac{e}{mc}\frac{\mathbf{B}}{\gamma}-\nabla\ln\gamma\times\mathbf{V}_p. \tag{2.8}$$

By the relationship

$$\nabla\left(\frac{n}{\gamma n_0}\right)=\frac{1}{\gamma}\nabla\left(\frac{n}{n_0}\right)-\frac{\nabla\gamma}{\gamma^2}\frac{n}{n_0}, \tag{2.9}$$

Eq. (2.4) can be rewritten as

$$\mathcal{Q}=k_p^2\nabla\left(\frac{n}{\gamma n_0}\right)\times\mathbf{P}_p-\frac{nk_p^2}{\gamma n_0}\mathbf{B}, \tag{2.10}$$

where $k_p=\omega_p/c$. Then the total magnetic field obeys the following evolution equation:

$$\nabla^2\mathbf{B}-\frac{\partial^2\mathbf{B}}{\partial\tau^2}-\frac{n}{\gamma n_0}\mathbf{B}=\sqrt{\frac{4\pi n_0}{m}}\nabla\left(\frac{n}{n_0\gamma}\right)\times\mathbf{P}_p, \tag{2.11}$$

where the dimensionless coordinates variables, $\rho=k_pr,\theta=k_p\varphi,\eta=k_pz,\tau=\omega_pt$, are used. $\nabla^2=\nabla_\perp^2+\nabla_\eta^2$, $\nabla_\perp^2$ in Eq. (2.11) denotes the Laplacian with the derivatives designated with respect to these variables (ρ) and (θ), and $\nabla_\eta^2=\partial^2/\partial\eta^2$.

By averaging over the fast laser frequency time scale but not over the plasma frequency, one obtains

$$\nabla^2\mathbf{b}_s-\frac{\partial^2\mathbf{b}_s}{\partial\tau^2}-\frac{1}{\gamma}\mathbf{b}_s=\frac{\omega_p}{\omega}\left\langle\nabla\left(\frac{n}{n_0\gamma}\right)\times\mathbf{p}_p\right\rangle, \tag{2.12}$$

where $\mathbf{p}_p=\mathbf{P}_p/mc$, $\mathbf{b}_s=e\mathbf{B}_s/mc\omega$, and $\mathbf{a}=e\mathbf{E}/mc\omega$.

Equation (2.12) is not only valid for the well-known ponderomotive mechanism and the inverse Faraday mechanism, but also for other mechanisms. It has no restriction for the laser parameters (i.e., polarization and intensities) and plasma characteristics (i.e., underdense and overdense plasma). The relativistic effect plays an important role in the induced magnetic field generation. The source term, $\mathbf{S}=\omega_p/\omega\langle\nabla(n/n_0\gamma)\times\mathbf{p}_p\rangle$, depends on the relativistic factor γ. In addition, the linear term $\mathbf{b}_s/\gamma$ of Eq. (2.12) also depends on γ. This implies that the relativistic effect significantly influences the magnitude, growth, and saturation of induced magnetic field generation. In the weakly relativistic regime $[(v/c)^2\ll 1]$, Eq. (2.12) has a form analogous to that presented in Refs. [20,21].

We approximate the radiation field $\mathbf{A}=mc^2\mathbf{a}/e$ as

$$\mathbf{a}=\tfrac{1}{2}a(x_\perp,z,t)(\hat{\mathbf{e}}_1+i\lambda\hat{\mathbf{e}}_2)\exp(i\psi)+\text{c.c.}, \tag{2.13}$$

where $a(x_\perp,z,t)$ is the real amplitude, slowly varying in time and space, λ is either equal to 1 or -1, corresponding to right and left circular polarization, respectively, $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ are the unit vector in the r and θ direction, and c.c. denotes the complex-conjugate terms. We can get the momentum,

$$\mathbf{P}_{p_\perp}=-\frac{i}{2}mca(\hat{\mathbf{e}}_1+i\lambda\hat{\mathbf{e}}_2)\exp(i\psi)+\text{c.c.}, \tag{2.14}$$

and the density,

$$n=-\frac{ican_0}{2\omega}(\hat{\mathbf{e}}_1+i\lambda\hat{\mathbf{e}}_2)\nabla(\gamma^{-1})\exp(i\psi)+\text{c.c.} \tag{2.15}$$

For a slowly varying laser pulse, we have $\partial^2b_s/\partial x_j^2\ll b_s,\partial^2b_s/\partial\tau^2\ll b_s$, where $j=r,\theta,z$. Combining Eqs. (2.12)–(2.15) gives approximately

$$\frac{\partial b_s}{\partial\rho}-\frac{\rho}{\gamma}b_s=-\frac{\rho}{\rho_0^2}\frac{\omega_p^2}{\omega^2}\frac{a^2}{\gamma^2}\left\{\left(1-\frac{a^2}{2\gamma^2}\right)+\left(\frac{5a^2}{2\gamma^2}-\frac{a^4}{\gamma^4}-1\right)\frac{\rho^2}{\rho_0^2}\right\}, \tag{2.16}$$

where $\rho_0=k_pR,R$ is the effective spot size of the laser. In the relativistic case, the induced magnetic field b_s approaches this level,

$$|b_s|\sim\frac{\omega_p^2}{\omega^2}. \tag{2.17}$$

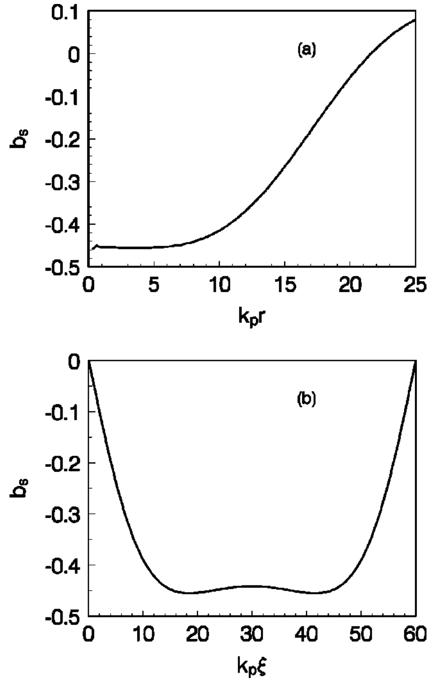

FIG. 1. Axial distribution of the magnetic field at the beam axis, $r=0$, and transverse distribution at $k_p\xi=20.0$. In bulk of the laser pulse, the magnetic field is almost a constant. The normalization vector potential $a_L=1.0$, the laser pulse duration is $k_pL=60$, and $\omega_p/\omega=0.09$.

The numerical solution of Eq. (2.12) gives the axial and radial variation of the induced magnetic field (see Fig. 1). For a long laser pulse, the variation of b_s with variables ξ is weak, where $\xi=t-z/V_g$, V_g is the group velocity of the laser pulse in plasma. In the bulk of the laser pulse, the induced magnetic field is almost a constant in the long laser pulse regime. So the magnetic field can be treated approximately as a constant magnetic field without variation with time in the following procedure. The quasistatic magnetic field may reach to the mega-Gauss level. For example, in the constant region of Fig. 1(b), the corresponding induced magnetic field is $B_s=7.65$ MG for the laser with frequency $\omega=3.0\times10^{14}$ rad/s.

III. RELATIVISTIC DYNAMICS OF THE INJECTED ELECTRON

A. Basic equations of electron motion

An injected electron with velocity $\mathbf{V}$ in an intense laser pulse can be described by the relativistic momentum equation,

$$\frac{d\mathbf{P}}{dt}=-e\left(\mathbf{E}+\frac{1}{c}\mathbf{V}\times\mathbf{B}\right), \tag{3.1}$$

where $\mathbf{P}$ is the momentum of the injected electron, $\mathbf{E}=\mathbf{E}_L+\mathbf{E}_s$, $\mathbf{B}=\mathbf{B}_L+\mathbf{B}_s$, $\mathbf{E}_L$ and $\mathbf{B}_L$ are the electric field and the magnetic field of the laser, respectively, $\mathbf{B}_s$ is the induced magnetic field, and $\mathbf{E}_s$ is the statically electric field. It can be obtained by Possion's equation,

$$\nabla\cdot\mathbf{E}_s=4\pi e(n_0-n), \tag{3.2}$$

where n_0 and n are, respectively, the background plasma density and the plasma electron density. When the laser pulse width L and the effective radius R are, respectively, larger than the plasma wavelength λ_p, i.e., $L\gg\lambda_p$ and $R\gg\lambda_p$, the effect of the statically electric field is so small that it can be neglected. In the remainder of this paper, we only consider the ultraintense laser electron interaction in the long laser pulse regime.

Consider an axially symmetric laser pulse. In cylindrical coordinates, the fields of laser and the induced magnetic field are expressed by

$$\mathbf{B}=(B_0\cos(\psi),B_0\sin(\psi),B_s), \tag{3.3a}$$

$$\mathbf{E}=(E_0\sin(\psi),-E_0\cos(\psi),E_s), \tag{3.3b}$$

where $\psi=\omega t-kz$, ω and k are the laser frequency and the wave number, respectively, and B_0,E_0 are, respectively, the amplitude of the electric field and the magnetic field of the laser wave.

Equation (3.1) can be rewritten as three component equations,

$$\frac{dP_r}{dt}+\omega_cP_\theta=-mca(\omega-k\dot{z})\sin\psi, \tag{3.4a}$$

$$\frac{dP_\theta}{dt}-\omega_cP_r=mca(\omega-k\dot{z})\cos\psi, \tag{3.4b}$$

$$\frac{dP_z}{dt}=-\omega_c\frac{B_0}{B_s}[P_r\sin(\psi)-P_\theta\cos(\psi)], \tag{3.4c}$$

where the dot over z denotes derivation versus time, ω_c is defined as the cyclotron frequency,

$$\omega_c=\frac{eB_s}{mc\gamma_e}=\frac{eB_sc}{\varepsilon}, \tag{3.5}$$

and ε is the energy of the injected electron, which satisfies

$$\frac{d\varepsilon}{dt}=-e\mathbf{V}\cdot\mathbf{E}. \tag{3.6}$$

Supposing a slowly varying laser pulse, the Maxwell equation gives approximately

$$\mathbf{k}\times\mathbf{E}=\frac{\omega}{c}\mathbf{B}. \tag{3.7}$$

Combining Eqs. (3.1), (3.6), and Eq. (3.7), we obtain

$$\frac{d\varepsilon}{dt}=\frac{\omega}{k}\frac{dP_z}{dt}=-\frac{eB_sc}{\omega_c^2}\frac{d\omega_c}{dt}. \tag{3.8}$$

Equation (3.8) expresses the fact that the laser wave cannot change the energy of the electron without also changing its momentum, a relationship easily understood if one adopts a photon picture of the interaction. It can be found that the energy of the injected electron is proportional to the longitudinal component of the momentum. Inserting Eq. (3.8) into Eqs. (3.4) gives

$$\frac{dp_r}{dt}+\omega_c p_\theta=-a(\omega-k\dot{z})\sin\psi, \tag{3.9a}$$

$$\frac{dp_\theta}{dt}-\omega_c p_r=a(\omega-k\dot{z})\cos\psi, \tag{3.9b}$$

$$\frac{d\omega_c}{dt}=\frac{\omega_c^3}{\omega}\frac{a}{b_s}[p_r\sin(\psi)-p_\theta\cos(\psi)]. \tag{3.9c}$$

B. Acceleration of injected electron

For investigating the acceleration of the injected electron, we try to obtain the energy equation. Defining

$$\sigma(t)=\int_0^t d\tau\omega_c(\tau), \tag{3.10}$$

Eq. (3.9a) and Eq. (3.9b) are equivalent to the integral equations

$$p_r=p_{\perp 0}\cos[\sigma(t)+\alpha_0]-a\int_0^t d\tau[(\omega-k\dot{z}(\tau)]\sin\varphi, \tag{3.11a}$$

$$p_\theta=p_{\perp 0}\sin[\sigma(t)+\alpha_0]+a\int_0^t d\tau[(\omega-k\dot{z}(\tau)]\cos\varphi, \tag{3.11b}$$

where $\varphi=\sigma(t)-\sigma(\tau)+\omega\tau-kz(\tau)$ and $p_{\perp 0}$ is the normalized initial transverse momentum at $t=0$, which satisfies $p_{r0}^2+p_{\theta 0}^2=p_{\perp 0}^2$, the constant α_0 given by $\tan\alpha_0=p_{r0}/p_{\theta 0}$. Inserting Eqs. (3.11) into Eq. (3.9c), gives

$$\frac{\dot{\omega}_c}{\omega_c^3}=\frac{a}{b_s^2\omega}\left\{p_{\perp 0}\sin(\Phi_1)+a\int_0^t d\tau[(\omega-k\dot{z}\tau)]\cos(\Phi_2)\right\}, \tag{3.12}$$

where $\Phi_1=\omega t-kz(t)-\sigma(t)-\alpha_0$ and $\Phi_2=\omega t-kz(t)-\sigma(t)-\omega\tau+kz(\tau)+\sigma(\tau)$. The phase satisfies the following equation:

$$\frac{d(\omega-kv_z)}{dt}=\frac{kc}{eB_s}\frac{d(\omega_c P_z)}{dt}. \tag{3.13}$$

Solving Eq. (3.13) gives

$$\omega-kv_z-\omega_c=d_1\omega_c+d_2\omega, \tag{3.14}$$

where $d_1=(\eta^2\omega-kv_{z0}-\omega_{c0})/\omega_{c0}$, $d_2=1-\eta^2,\eta=B_0/E_0$. We note that the Doppler frequency is induced by the longitudinal velocity component v_z of the injected electron and the phase depends on the induced magnetic field and the refractive index η.

The resonance condition for an electron with a velocity component in the longitudinal direction is

$$\omega-kv_z-\omega_c=0. \tag{3.15}$$

It can be rewritten as

$$\Omega=-\frac{d_2}{d_1}\omega. \tag{3.16}$$

This condition is satisfied by two different cases. (i) The injected electron is at resonance at $t=0$, i.e., $\omega-kv_{z0}-\omega_{c0}=0$, and the laser field satisfies $E_0=B_0$. (ii) The induced magnetic field satisfies

$$b_s=\eta^2(1-\gamma)+\gamma-\eta v_0, \tag{3.17}$$

where $v_0=u_{z0}/c$. The subscript 0 denotes the initial values.

Inserting Eq. (3.14) into Eq. (3.12) gives

$$\frac{\dot{\omega}_c}{\omega_c^3}=\frac{a}{b_s^2\omega}\Big\{p_{\perp 0}\sin[d_1\sigma(t)+d_2\omega t-\delta_0]+a\int_0^t d\tau[(d_1+1)\omega_c(\tau)+d_2\omega]\cos[d_1\sigma(t)+d_2\omega t-d_1\sigma(\tau)-d_2\omega\tau]\Big\}, \tag{3.18}$$

where $\delta_0=kz_0+\alpha_0+\pi/2$. Multiplication by $d_1\omega_c(t)+d_2\omega$ and integrating Eq. (3.12) from 0 to t gives

$$\frac{d_1}{\omega_c}+\frac{d_2\omega}{2\omega_c^2}=\frac{ap_{\perp 0}}{b_s^2\omega}\sin[d_1\sigma(t)+d_2\omega t-\delta_0]+\frac{a}{b_s^2\omega}\int_0^t d\tau[(d_1+1)\omega_c(\tau)+d_2\omega]\sin[d_1\sigma(t)+d_2\omega t-d_1\sigma(\tau)-d_2\omega\tau]+\frac{d_1}{\omega_{c0}}+\frac{d_2\omega}{2\omega_{c0}}+\frac{ap_{\perp 0}\sin\delta_0}{b_s^2\omega}. \tag{3.19}$$

If we now differentiate Eq. (3.18) with respect to time and subtract the resulting equation from $d_1\omega_c+d_2\omega$ times Eq. (3.19), we have

$$\frac{d^2}{dt^2}\left(\frac{1}{2\omega_c^2}\right)+\frac{d_2^2\omega^2}{2\omega_c^2}+\frac{3d_1d_2\omega}{2\omega_c}-N_0\omega_c+M_0=0,$$

$$M_0=d_1^2-\frac{d_1d_2\omega}{\omega_{c0}}-\frac{d_2^2\omega^2}{2\omega_{c0}^2}-\frac{d_2\omega ap_{\perp 0}\sin\delta_0}{b_s^2\omega}-\frac{a^2d_2}{b_s^2},$$

$$N_0=\frac{d_1^2}{\omega_{c0}}+\frac{d_1d_2\omega}{2\omega_{c0}^2}+\frac{d_1\omega ap_{\perp 0}\sin\delta_0}{b_s^2\omega}+(d_1+1)\frac{a}{b_s^2\omega}. \tag{3.20}$$

After appropriate calculation, one obtains the energy equation of the injected electron,

$$\frac{d^2\varepsilon^2}{dt^2}+d_2^2\omega^2\varepsilon^2+3eB_scd_1d_2\omega\varepsilon-\frac{2(eB_sc)^3N_0}{\varepsilon}+2(eB_sc)^2M_0=0. \tag{3.21}$$

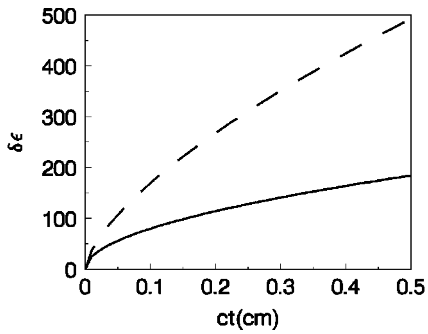

FIG. 2. The evolution of energy of the injected electron with time. $\delta_0=\pi/3$, $\beta_0=0.9$, $a_L=2.0$, $\omega=6.0\times10^{14}$ rad/s, $\omega_p/\omega=0.01$ (solid), and $\omega_p/\omega=0.1$ (dashed).

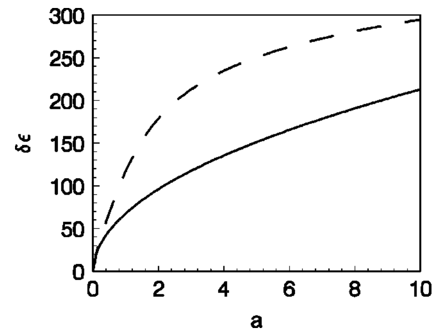

FIG. 3. The energy rate $\delta\epsilon=\varepsilon/\varepsilon_0$ is plotted as a function of laser intensities. The more intense laser is more efficient for electron acceleration. The parameters are $\delta_0=\pi/3$, $ct=0.15$ cm, $\omega=6.0\times10^{14}$ rad/s, $\beta_0=0.5$, $\omega_p/\omega=0.01$ (solid), and $\omega_p/\omega=0.1$ (dashed).

If $E_0=B_0$, and the cyclotron-resonance condition is satisfied at $t=0$, Eq. (3.21) gives

$$\left(\frac{d\varepsilon}{dt}\right)^2-\omega^2\varepsilon_0^2\left\{2h_1\frac{\varepsilon_0}{\varepsilon}+(h_2^2-2h_1)\left(\frac{\varepsilon_0}{\varepsilon}\right)^2\right\}=0, \tag{3.22}$$

where $h_1=-ab_s$ and $h_2=\beta_0 a\cos\delta_0$. It is easy to obtain the injected electron energy,

$$6h_1^2\omega t=u^3-3(h_2^2-2h_1)u+2h_2(h_2^2-3h_1), \tag{3.23}$$

where $u=\sqrt{2h_1(\varepsilon-\varepsilon_0)/\varepsilon_0+h_2^2}$, $\beta_0=V_{\perp 0}/c$, and $V_{\perp 0}$ is the transverse component of the initial velocity.

Equation (3.23) expresses time as a function of energy. In order to invert it and express energy as a function of time, a cubic equation must be solved. While it is possible to do this in closed form, the resulting expression is quite involved and will not be given here. The asymptotic form of the solution as $t\to\infty$ is quite simple, however, and can be directly obtained from Eq. (3.22),

$$\frac{\varepsilon^*}{\varepsilon_0}\sim\left(\frac{9a^2}{2}\frac{\omega_p^2}{\omega^2}\right)^{1/3}(\omega t)^{2/3}=\left[\frac{9}{2}(k_pL)^2a^2\left(\frac{T}{T_R}\right)^2\left(\frac{T_R}{\tau_L}\right)^2\right]^{1/3}, \tag{3.24}$$

where τ_L is the laser pulse duration, $T_R=kR^2/2c$ is the Rayleigh time, and T is the accelerating time of an electron in the combination field. It is easy to keep the right-hand side of Eq. (3.24) larger than unity, so Eq. (3.24) demonstrates that the injected electron obtains energy from the laser pulse.

As an example, we consider an intense laser beam propagating in an underdense plasma. The following parameters were chosen: $a=0.33$, $k_pL=60$, and $k_pR=10$. It has been illustrated that, in a preplasma leaky channel, by $20T_R$ the pulse keeps the same smooth shape as the initial one but with a slightly reduced amplitude [28]. So in our case, taking the propagation time $T=20T_R$, then the energy rate $\delta\epsilon^*$ $(=\varepsilon^*/\varepsilon_0)$ of the injected electron can be estimated, i.e., $\delta\epsilon^*\sim10^2$. If the initial energy of the injected electron $\varepsilon_0=1$ keV, the energy gain of the electron from the laser pulse is $\varepsilon^*\sim100$ keV. If we use the energy obtained as the initial energy, and let the electron inject another combination field, the final energy of the injected electron may reach 10 MeV.

Numerical evaluation has been performed for an injected electron in the following. Eqs. (2.12) and (3.22) are solved for an initial radiation pulse of the form $a=a_0\sin(\pi/L)\xi\exp(-r^2/R_0^2)$ with $k_pL=60$, $k_pR_0=10$. The energy of the injected electron increases with time. Figure 2 shows that the more dense plasma density is more effective for the electron acceleration. The dependence of the energy of the injected electron on several parameters is plotted in Figs. 3–5. One sees that the more intense laser enhances the electron acceleration; the asymptotic expression (3.24) is apparently in agreement with these results. The energy of the injected electron is associated with the initial velocity $v_{\perp 0}$ and the initial angle δ_0. With the increase of ω_p/ω, however, the dependence of energy on the $v_{\perp 0}$ and δ_0 becomes weak. Especially, for the plasma with $\omega_p/\omega\to1$, the effects of δ_0 and β_0 are so weak that these effects can be neglected.

Consider a special case, if $\beta_0\cos^2\delta_0=2ab_s$, which is equivalent to the condition $a=\beta_0/2b_s\cos^2\delta_0$, $h_2^2-2h_1=0$. Equation (3.21) gives

$$\varepsilon=\varepsilon_0\left[1+\left(\frac{9h_1}{2}\right)^{1/2}\omega t\right]^{2/3}. \tag{3.25}$$

We note that, with the enhancing of the induced magnetic field, the energy of the injected electron increases. In addition, the longer accelerating time is beneficial to the electron acceleration.

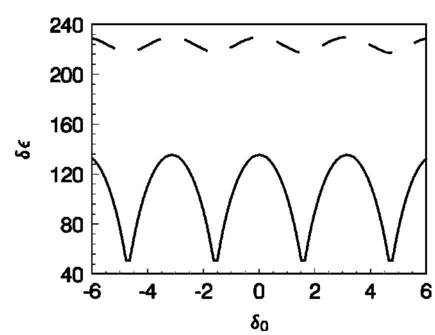

FIG. 4. The energy rate $\delta\epsilon$ is plotted as a function of initial angle δ_0. The more dense plasma density is, the more weak is the dependence of energy on δ_0. The parameters are $ct=0.15$ cm, $\omega=6.0\times10^{14}$ rad/s, $\beta_0=0.5$, $a=2.0$, $\omega_p/\omega=0.01$ (solid), and $\omega_p/\omega=0.1$ (dashed).

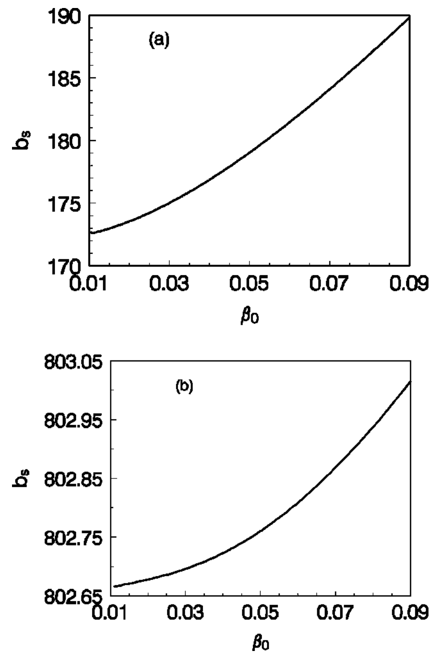

FIG. 5. The energy rate $\delta\epsilon$ is plotted as a function of initial normalized velocity β_0. The more dense plasma density is, the more weak is the dependence of energy on β_0. The parameters are $ct=0.15$ cm, $\omega=6.0\times10^{14}$ rad/s, $\delta_0=\pi/3$, $a=2.0$, $\omega_p/\omega=0.01$ (a), and $\omega_p/\omega=0.1$ (b).

However, the interaction of the injected electron with the combination field is terminated, as the cyclotron radius of the injected electron is larger than the effective spot size of the laser beam, or the injected electron escapes from the laser pulse due to the dephasing in the longitudinal direction. We study the accelerating time in the following.

C. Termination of electron acceleration

1. The general case

In the cyclotron-resonance condition, the phase of the injected electron satisfies

$$\frac{d\phi}{dt}=\frac{eB_s c}{\varepsilon}. \tag{3.26}$$

Equations (3.11) and Eq. (3.9c) give the components of the momentum as a function of ϕ,

$$p_r=p_{\perp 0}\cos(\phi+\alpha_0)-a\phi\sin(\phi), \tag{3.27a}$$

$$p_\theta=p_{\perp 0}\sin(\phi+\alpha_0)+a\phi\cos(\phi), \tag{3.27b}$$

$$p_z=p_{z0}+\eta(\varepsilon'-\varepsilon_0'). \tag{3.27c}$$

For convenience, we express the energy as a function of phase. By the relativistic energy-momentum transfer relationship, $\varepsilon^2=m^2c^4+P^2c^2$, we have

$$(1-\eta^2)w^2-2h_s w=f(\phi), \tag{3.28}$$

where $w=\varepsilon'-\varepsilon_0', h_s=(p_{z0}\eta-\varepsilon_0')$, $f(\phi)=a^2\phi^2+p_{\perp 0}a\phi\sin\alpha_0-2\varepsilon_0'^2$, ε' is the normalized energy, and $\varepsilon'=\varepsilon/mc^2$.

The injected electron escapes from the combination field in two ways. (i) The injected electron spirals outward when the cyclotron radius is larger than the laser beam radius. (ii) The injected electron dephases with the laser wave in the longitudinal direction. To demonstrate these two physical processes, we calculate the critical phase and the accelerating time in the following.

First, we assume the injected electron is in phase with the laser wave in the longitudinal direction, which needs the relationship

$$V_z=V_g, \tag{3.29}$$

where $V_g=c(1-\omega_p^2/2\omega^2\gamma)$ and $V_z(\phi)=cp_z/\varepsilon'$. Equation (3.29) gives

$$\frac{\gamma_e}{\gamma}=\frac{2(1-\eta p_{z0}/\varepsilon_0')}{\omega_p^2/2\omega^2-(1-\eta)\gamma}. \tag{3.30}$$

Keeping the injected electron in the laser beam requires the condition

$$k_p r\leqslant k_p R_0, \tag{3.31}$$

where r is the cyclotron radius of the injected electron, given by $r=r_{c0}\gamma_e$. After simple derivation, the critical phase is given by

$$\phi_{tc}=\frac{\pm\sqrt{p_{\perp 0}^2\sin^2\alpha_0+8(\varepsilon_0'^2+\ell^2)}-p_{\perp 0}\sin\alpha_0}{2a}, \tag{3.32}$$

where $\ell=\frac{1}{2}(R_0/r_{c0}-\varepsilon_0')^2(1-\eta^2)-h_s(R_0/r_{c0}-\varepsilon_0')$. The injected electron spirals outward from the laser beam, as $\phi>\phi_c$. The accelerating time of the injected electron is determined by

$$T_t=\frac{R_0}{r_{c0}}\,\frac{\phi_{tc}-\phi_0}{b_s w}. \tag{3.33}$$

Next, consider the dephasing of the injected electron with laser wave in the longitudinal direction. Indeed, this condition $V_z=V_g$ is very restricted, because it needs a much larger electron energy. Equation (3.30) shows $\gamma_e/\gamma\to 4\omega^2/\omega_p^2$, as $\eta\to 1$. For a plasma with $\omega_p/\omega=0.01$, $\gamma_e\sim 10^4$. So the dephasing of the injected electron in the longitudinal direction is more important. In this case, the accelerating time is given by

$$T_l=\frac{\tau_L}{\Delta-\sigma}, \tag{3.34}$$

where $\Delta=\eta-\beta_g$, $\beta_g=V_g/c$, $\sigma=(\eta\varepsilon_0'-p_{z0})/\varepsilon_c'$, and ε_c' is the normalized energy at the time of the electron escaping from the laser pulse. Because ε_c' is a function of T_l, i.e., $\varepsilon_c'=f(T_l)$, Eq. (3.34) is a nonlinear algebraic equation of T_l. As $\sigma\to 0$, Eq. (3.34) can be expressed by

$$T_l=\frac{\tau_L}{\eta-\beta_g}. \tag{3.35}$$

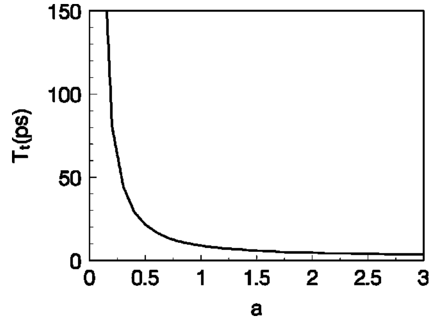

FIG. 6. The accelerating time T_t as a function of laser intensities. $R_0=15$ μm, $\omega=6.0\times10^{14}$ rad/s, $\omega_p/\omega=0.01$.

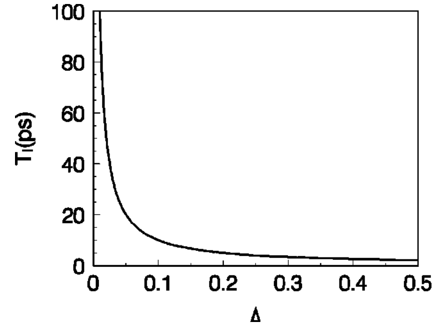

FIG. 7. The accelerating time T_l as a function of $\Delta=\eta-\beta_g$. $\eta-p_{z0}/\varepsilon_0'=0.8$, $\tau_L=1$ ps.

It illustrates that the more underdense plasma density is more effective for the accelerating time T_l. It may become much larger as Δ approaches zero.

2. The special case

For demonstrating and clearly showing the physical process, we only consider now a special case, in which the energy of the injected electron is given by Eq. (3.25). In the cyclotron resonance-condition, Eq. (3.26) may be rewritten as

$$\frac{d\phi}{dt}=\frac{b_s\omega H^2}{\varepsilon_0'(\phi-H)^2}, \tag{3.36}$$

where $H=\sqrt{(2|b_s|/a)}/\varepsilon_0'$ and ϕ_0 is the phase at $t=0$. In the following calculation, we take $\phi_0=0$. It is easy to obtain

$$V_z(\phi)=c\left\{1-\frac{H^2}{\varepsilon_0'}h^*(\phi-H)^{-2}\right\}, \tag{3.37}$$

where $h^*=\varepsilon_0'\eta-p_{z0}$. The relationship $V_g=V_z$ is valid with the condition (3.30), and the injected electron is in phase with the laser pulse.

The cyclotron radius can be expressed by the phase ϕ,

$$r=r_{c0}\varepsilon_0'(\phi-H)^2/H^2, \tag{3.38}$$

where r_{c0} is the initial radius of the injected electron. The cyclotron radius becomes large with time. When $r\geqslant R_0$, which is equivalent to the condition

$$\phi_c\geqslant H\varepsilon_0'+\sqrt{\frac{2|b_s|}{a}\frac{R_0}{r_{c0}}}=\sqrt{\frac{2b_s}{a}}\left[\left(\frac{R_0}{r_{c0}}\right)^{1/2}+1\right], \tag{3.39}$$

the electron spirals outward from the laser beam. The critical phase is directly influenced by the laser and the induced magnetic field. The time of the injected electron in the laser pulse is given by

$$T_t=\frac{1}{3\omega}\sqrt{\frac{2}{h_1}}\left[\left(\frac{R_0}{r_{c0}}\right)^{3/2}+1\right]. \tag{3.40}$$

The accelerating time T_t as a function of laser intensities is plotted in Fig. 6.

By Eq. (3.34) and Eq. (3.25), the accelerating time in the longitudinal direction is given by

$$\left(\Delta-\frac{\tau_L}{T_l}\right)\gamma_{ec}=\eta-\frac{p_{z0}}{\varepsilon_0'}, \tag{3.41}$$

where $\gamma_{ec}=[1+(9h_1/2)^{1/2}\omega T_l]^{2/3}$. The accelerating time as a function of Δ is plotted in Fig. 7. Noting $\gamma_{ec}\gg1$, Eq. (3.41) gives $T_l\simeq\tau_L/\Delta$. It has nothing to do with the laser intensities.

IV. TRAJECTORY OF THE INJECTED ELECTRON

The velocity components of the injected electron can be expressed by

$$V_r(\phi)=\frac{cp_r}{\varepsilon'},\quad V_\theta(\phi)=\frac{cp_\theta}{\varepsilon'},\quad V_z(\phi)=\frac{cp_z}{\varepsilon'}. \tag{4.1}$$

It is straightforward to show that

$$\frac{d(k_px_r)}{d\phi}=\frac{\omega_p}{b_s\omega}p_r, \tag{4.2a}$$

$$\frac{d(k_px_\theta)}{d\phi}=\frac{\omega_p}{b_s\omega}p_\theta, \tag{4.2b}$$

$$\frac{d(k_px_z)}{d\phi}=\frac{\omega_p}{\omega b_s}(p_{z0}+\eta w). \tag{4.2c}$$

Subsequently, the trajectory of the injected electron is described by

$$k_px_r=\frac{\omega_p}{b_s\omega}\{p_{\perp0}[\sin(\phi+\alpha_0)-\sin(\alpha_0)] -a(\sin\phi-\phi\cos\phi)\}, \tag{4.3a}$$

$$k_px_\theta=-\frac{\omega_p}{b_s\omega}\{p_{\perp0}[\cos(\phi+\alpha_0)-\cos(\alpha_0)) -a(1-\cos\phi-\phi\sin\phi)\}, \tag{4.3b}$$

$$k_px_z=\frac{\omega_p}{\omega b_s}\left\{p_{z0}(\phi-\phi_0)+\eta\int_{\phi_0}^{\phi}w\,d\phi\right\}. \tag{4.3c}$$

Equations (4.3a) and (4.3b) show that the variation of the transverse coordinates with phase has nothing to do with the form of the energy of the injected electron, but the trajectory of the injected electron depends on the induced magnetic field. Due to the induced magnetic field dependence on the laser intensities, we can control the electron motion by the laser intensities.

In the special case, the velocity of the injected electron is given by

$$V_r(\phi)=\frac{cH^2}{\varepsilon_0'}\left\{p_{\perp 0}\frac{\cos(\phi+\alpha_0)}{(\phi-H)^2}-a\frac{\phi\sin(\phi)}{(\phi-H)^2}\right\}, \tag{4.4a}$$

$$V_\theta(\phi)=\frac{cH^2}{c}\left\{p_{\perp 0}\frac{\sin(\phi+\alpha_0)}{(\phi-H)^2}+a\frac{\phi\cos(\phi)}{(\phi-H)^2}\right\}, \tag{4.4b}$$

$$V_z(\phi)=c\left\{1-\frac{H^2}{\varepsilon_0'}h^*(\phi-H)^{-2}\right\}. \tag{4.4c}$$

It is straightforward to obtain the following expressions:

$$\frac{dk_px_r}{d\phi}=\frac{\omega_p}{b_s\omega}[p_{\perp 0}\omega_{c0}\cos(\phi+\alpha_0)-a\phi\sin\phi], \tag{4.5a}$$

$$\frac{dk_px_\theta}{d\phi}=\frac{\omega_p}{b_s\omega}[p_{\perp 0}\omega_{c0}\sin(\phi+\alpha_0)+a\phi\cos\phi], \tag{4.5b}$$

$$\frac{dk_px_z}{d\phi}=\frac{\omega_p}{b_s\omega}\left(p_{z0}+\eta\frac{\varepsilon_0'}{H^2}[(\phi-H)^2-H^2]\right). \tag{4.5c}$$

The transverse coordinates k_px_r and k_px_θ of the electron keep the same form as Eq. (4.3a) and Eq. (4.3b), respectively, and k_px_z may be expressed as

$$k_px_z=\frac{\omega_p}{b_s\omega}\left\{h^*\phi+\eta\varepsilon_0'H+\frac{\eta\varepsilon_0'(\phi-H)^3}{3H^2}\right\}. \tag{4.6}$$

When $\phi=\phi_c$, the injected electron will exit the combination field with a certain angle ϑ, where ϑ is the angle between the propagation direction of the laser and the emission direction of injected electron. One can easily obtain

$$\tan\vartheta=\frac{\sqrt{p_{\perp 0}^2+a^2\phi_c^2}}{p_{z0}+\eta w_c}, \tag{4.7}$$

where w_c satisfies Eq. (3.28) and takes the value at $\phi=\phi_c$. By Eq. (3.28), one can get $w_c\approx(a^2\phi_c^2+p_{\perp 0}a\phi_c\sin\alpha_0-2\varepsilon_0'^2)/(\varepsilon_0'-p_{z0}\eta)$ as $\eta\to 1$. When $a\phi_c>\frac{1}{2}\sqrt{A^2+8\varepsilon_0'^2}-\frac{1}{2}A, w_c>a\phi_c$, where $A=h_s+p_{\perp 0}\sin\alpha_0$. In addition, $p_{z0},p_{\perp 0}^2\ll a^2\phi_c^2$. Consequently, the range of the emission angle is $0<\vartheta<\pi/4$. At $t=0$, the incidence angle is $\tan\vartheta_i=p_{\perp 0}/p_{z0}$, obviously $\vartheta<\vartheta_i$.

V. REMARKS AND CONCLUSIONS

For an efficiently accelerating electron, optical guiding is needed. When an ultraintense laser pulse propagates in a plasma, it is subject to various limitations [29] placed on the acceleration distance, such as diffraction, phase detuning distance, laser depletion, and various instabilities, i.e., Raman instabilities [29,30]. These effects strictly shorten the propagation distance, and subsequently the accelerating time. Fortunately, these limitations may be overcome or partly avoided by the optical guiding. It has been found that the relativistic guiding is efficient for a long intense laser pulse when the power of the laser is greater than the critical power, $P(GW)\geq 17.4(\omega^2/\omega_p^2)$ [31], and the plasma channel can effectively guide the ultraintense short laser pulse. Experimental results have confirmed these predictions [32,33]. A recent experiment demonstrates that the ultraintense laser pulse can be guided for a long distance, i.e., 2.2 cm [33].

The transverse gradient of the induced magnetic field causes a transverse drift velocity of the injected electron,

$$\mathbf{V}_{\perp s}=(\mathbf{B}_s\times\nabla\mathbf{B}_s)(\tfrac{1}{2}v_c^2+v_\parallel^2)/B_s\omega_{c0}, \tag{5.1}$$

where v_c is the cyclotron velocity of the electron and $v_\parallel$ is the drift velocity of the electron in the longitudinal direction. However, the gradient of the magnetic field is so small for a wide laser beam on the scale of the accelerated electron that this effect can be neglected. It is noticeable that due to negative magnetic field, the transverse drift velocity confines the injected electron radially.

The above theory is valid in the bulk of the laser pulse, and is valid for the plane electromagnetic wave. However, the validity is deviated in the edge of the laser pulse due to the magnetic field dependence on the coordinates $\xi(=t-z/v_g)$. In this region, the magnetic field varies in the direction of the laser propagation with time. A more accurate theory will be necessary.

In conclusion, the relativistic dynamics of the injected electron subjected to a combination field of an electromagnetic field of an intense circularly polarized laser pulse along with the quasistatic magnetic field produced by the inverse Faraday effect is investigated theoretically. The injected electron is accelerated by the like-ECR mechanism: energy gains of injected electrons can be obtained by Doppler-shifted cyclotron resonance in the combination field for the appropriate injection velocity. This IFE magnetic field also acts to confine these electrons radially and on exiting the plasma adiabatically. At a special case, the emission angle of injected electrons $\vartheta<\pi/4$ and $\vartheta<\vartheta_i$, it is in this way that the transverse electron energy is converted to axial energy. There are two limits to the energy gain: (i) the cyclotron radius of the energetic electrons becoming comparable to the beam, and (ii) axial dephasing, but the latter limit is more important. These limits depend on the initial parameters of the laser and the injected electron, as well as the propagation length of the laser pulse in plasma. The induced magnetic field influences significantly the motion of the electron. Due to the induced magnetic field dependence on the laser intensities, one can control the electron motion by adjusting the laser intensities.

[1] L.F. Mollenauer, R.H. Stolen, and J.P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[2] H.A. Haus, J.G. Fujimoto, and E.P. Ippen, IEEE J. Quantum Electron. **QE-28**, 2086 (1992).
[3] B. Luther-Davies *et al.*, Kvant. Elektron. (Moscow) **19**, 317 (1992) [Sov. J. Quantum Electron. **22**, 289 (1992)].
[4] C.J. Joshi and P.B. Corkum, Phys. Today **47**(1), 36 (1995).
[5] G. Mourou and D. Umstadter, Phys. Fluids B **4**, 2315 (1992).
[6] M.D. Perry and G. Mourou, Science **264**, 917 (1994).
[7] F.V. Hartemann *et al.*, Phys. Rev. E **51**, 4833 (1995).
[8] F.V. Hartemann, Phys. Plasmas **5**, 2037 (1998).
[9] B. Quesnel and P. Mora, Phys. Rev. E **58**, 3719 (1998).
[10] C.J. McKinstrie and E.A. Startsev, Phys. Rev. E **54**, R1070 (1996).
[11] T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[12] C.E. Clayton *et al.*, Phys. Rev. Lett. **54**, 2343 (1985).
[13] P. Sprangle, E. Esarey, and J. Krall, Phys. Plasmas **3**, 2183 (1996).
[14] A. Ting *et al.*, Phys. Rev. Lett. **77**, 5377 (1996); S.P. Le Blanc *et al.*, *ibid.* **77**, 5381 (1996).
[15] D. Umstadter, E. Esarey, and J. Kim, Phys. Rev. Lett. **72**, 1224 (1994); D. Umstadter *et al.*, Phys. Rev. E **51**, 3484 (1995).
[16] G. Bonnaud, D. Teychenne, and J.L. Bobin, Phys. Rev. E **50**, R36 (1994).
[17] S. Kawata, T. Maruyama, H. Watanabe, and I. Takahashi, Phys. Rev. Lett. **66**, 2072 (1991).
[18] M.S. Hussein and M.P. Pato, Phys. Rev. Lett. **68**, 1136 (1992).
[19] S.C. Wilks, W.L. Kruer, M. Tabak, and A.B. Landon, Phys. Rev. Lett. **69**, 1383 (1992); R.N. Sudan, *ibid.* **70**, 3075 (1993).
[20] L. Gorbunov, P. Mora, and T.M. Antonsen, Jr., Phys. Rev. Lett. **76**, 2495 (1996).
[21] V.K. Tripathi and C.S. Liu, Phys. Plasmas **1**, 990 (1994).
[22] J. Deschamps, M. Fitaire, and M. Lagoutte, Phys. Rev. Lett. **25**, 1330 (1970).
[23] G.A. Askar'yan, S.V. Bulanov, F. Pegoraro, and A.M. Pukhov, Pis'ma Zh. Éksp. Teor. Fiz. **60**, 240 (1994) [JETP Lett. **60**, 251 (1994)]; A. Pukhov and J. Meyer-ter-vehn, Phys. Rev. Lett. **76**, 3975 (1996).
[24] A.D. Steiger and C.H. Woods, Phys. Rev. A **5**, 1467 (1972).
[25] Z.M. Sheng and J. Meyer-ter-Vehn, Phys. Rev. E **54**, 1833 (1996).
[26] G. Zeng and X. He, Phys. Plasmas **6**, 2954 (1999).
[27] S.V. Bulanov, M. Lontano, T. Zh. Esirkepov, F. Pegoraro, and A.M. Pukhov, Phys. Rev. Lett. **76**, 3562 (1996).
[28] T.M. Antonsen, Jr. and P. Mora, Phys. Rev. Lett. **74**, 4440 (1995).
[29] T.M. Antonsen, Jr. and P. Mora, Phys. Fluids B **5**, 1440 (1993).
[30] P. Sprangle, C.M. Tang, and E. Esarey, IEEE Trans. Plasma Sci. **PS-15**, 145 (1987).
[31] P. Sprangle and E. Esarey, Phys. Fluids B **4**, 2241 (1992); S.V. Bulanov and N.M. Naumova, Phys. Plasmas **1**, 745 (1994); A.V. Borovskii, V.V. Korobkin, and A.M. Prokhorov, Zh. Éksp. Teor. Fiz. **106**, 148 (1994) [JETP **79**, 81 (1994)].
[32] A.B. Borisov *et al.*, Phys. Rev. Lett. **68**, 2309 (1992).
[33] C.G. Durfee III and H.M. Milchberg, Phys. Rev. Lett. **71**, 2409 (1993); C.G. Durfee III, J. Lynch, and H.M. Milchberg, Phys. Rev. E **51**, 2368 (1995).